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# Performance Analysis of a Multi-Carrier CDMA (MC-CDMA) Wireless Communication System in Presence of Carrier Frequency Jitter over Rayleigh Fading Channel

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**ABSTRACT**–Multi-Carrier CDMA (MC-CDMA) systems are highly promising for future broad band communication due to its inherent merits of security and flexible capacity. In this paper, we propose a MC-CDMA system with frequency diversity in transmission as well as space diversity in reception using multiple antennas with Maximal Ratio Combining (MRC) technique. An analogical approach is presented to evaluate the analytical expression of signal to noise plus Multiple Access Interference (MAI) considering frequency jitter over a Rayleigh fading channel. Results are evaluated numerically for different system parameters and improvement is observed in system BER due to diversity. The system model is numerically evaluated for number of antenna elements, number of simultaneous user, processing gain etc.

**KEYWORDS** – MC-CDMA, OFDM, CFO, ICI, SNIR, Frequency Jitter, INR.

## I. INTRODUCTION

Multicarrier code-division multiple access (MC-CDMA) is a promising technique to challenge the demand of high data rate in a wireless communication system. It represents the fusion of two distinct techniques viz OFDM and CDMA [7], OFDM being very efficient in combating multipath fading as well as Inter Symbol Interference (ISI). Unfortunately OFDM is very much sensitive to the synchronization errors, Carrier frequency offset (CFO), Phase Noise or timing jitter [10]. On the other hand severe Inter Chip-Interference (ICI) is the main drawback of CDMA. Orthogonal multicarrier modulation combined with CDMA has been proposed to solve this problem [3]. According to Ref.[3] it is also proved that multicarrier diversity is better than multipath diversity in terms of BER performance.

Performance analysis of random an OFDM WC system with random CFO jitter is reported earlier [10]. The performance of a MC-DS-CDMA system with OFDM multicarrier modulation is highly sensitive to the orthogonality among the OFDM subcarriers. The frequency offset due to fading and Doppler offset causes the OFDM sub carrier to loose orthogonality. As a consequence the performance of an OFDM link is highly degraded [10]. Recently, the effect of fading in Rayleigh and Nakagami-m channel on a MC-DS-CDMA system are reported [11,12]. It is very much important to find the effect of random frequency jitter on the performance of a multi carrier CDMA system in presence of a multipath fading. In this paper we developed the closed form SINR expression for MC-CDMA system and BER performance evaluated for different system parameters. The effect of frequency jitter is also evaluated numerically.

## II. MC-CDMA SYSTEM MODEL

### A. Transmitter Model

The transmitter model is shown in Fig. 1 where input data is a single bit  $b_j$  through copier distributed to  $L$  number of channel. Parallel data symbols are then coded by the particular PN sequence having chip duration  $T_c$  and processing gain  $N$ . Thereafter, each data symbol is spread in time domain. The coded symbol of each data stream is then modulated by sub-carrier of respective channel and finally they are combined and transmitted through OFDM modulator.

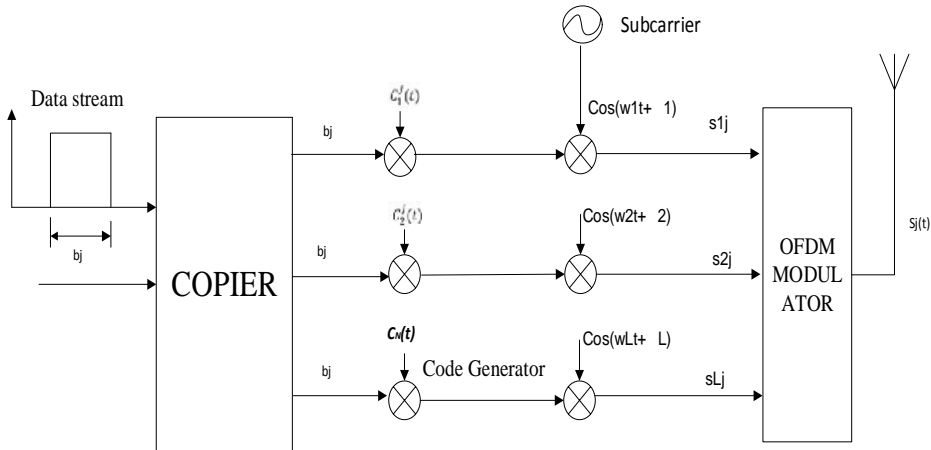


Fig. 1. Block Diagram of a MC-CDMA Transmitter

### B. Receiver Model

Receiver of the  $j^{\text{th}}$  user received by  $M$  number of antennas as shown in Fig.2. Each received signal will be converted into parallel data through power splitter. Receiver block is mainly consists of coherent detector having a low pass filter (LPF), MRC, decision circuit, EGC and Power Splitter. Coherent detector exploits the knowledge of carriers phase to detect the signals. EGC weights each signal branch with the same factor irrespective of the signal amplitude.

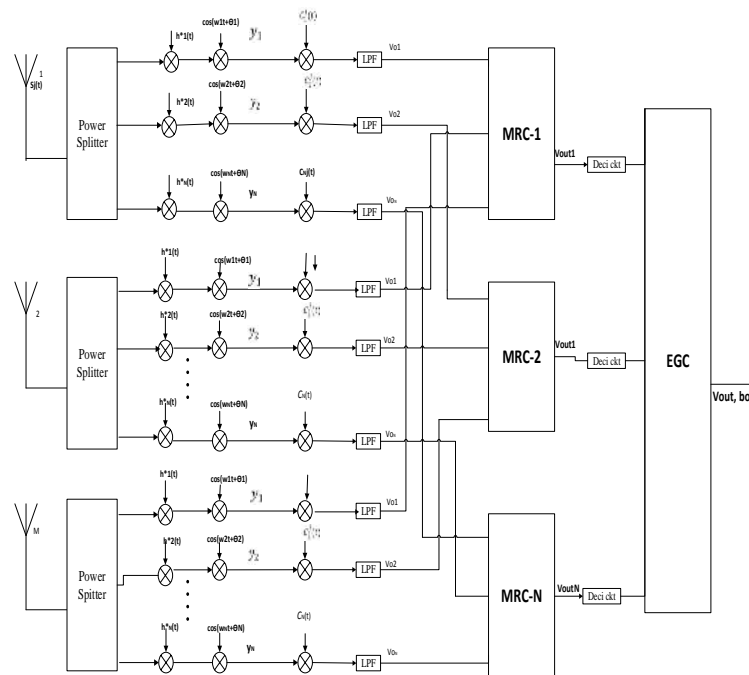


Fig. 2. Block Diagram of a MC-CDMA Receiver

### III. THEORETICAL ANALYSIS OF A MC-CDMA SYSTEM

Assuming, a total K number of users in the system and perfect power control are employed, the general expression of the transmitted signal of the  $S_j^{\text{th}}$  user with code  $C^j(t)$  and Subcarrier are given as:

$$s_i^j = b^j \cdot c_i^j \cdot \cos(w_i t + \theta_i) \tag{1}$$

Where,  $i=1: N$ ,  $N$  is the Code Length (Processing Gain) which is equal to number of sub carrier. Considering Binary PSK (BPSK) modulation, the output for  $j^{\text{th}}$  modulator can be expressed as:

$$s^j(t) = \sum_{i=1}^N s_i^j(t) \tag{2}$$

$$s^j(t) = \sum_{i=1}^N b^j \cdot c_i^j \cdot \cos(w_i t + \theta_i(t)) \tag{3}$$

Here,  $b^j$  is transmitted symbol of  $j^{\text{th}}$  user in  $i^{\text{th}}$  sub channel which is being modulated by one of the  $N$  sub-carrier,  $C_i^j$  is the PN sequence for  $j^{\text{th}}$  user,  $i = [1 2... N]$  is the number of sub-carriers in the system,  $w$  is the instantaneous/angular velocity of each sub-carrier assumed to be uniformly distributed over  $[0, 2\pi]$  and  $\theta(t)$  is considered as frequency jitter.

Total  $L$  number of antennas are considered in the system. Each of them are having Coherent receiver with  $N$  branches. Receiver of the  $j^{\text{th}}$  user receives all the signal transmitted by  $K$  number of users. Thus the expression of received signal corresponding to  $v$  path is:

$$r_i^j(t) = s_i^j(t) \cdot H^j + n(t) \tag{4}$$

$$r_i^j(t) = s_i^j(t) \left[ h_i(t) \cdot h_i^*(t) \right] + n(t)$$

$$r^j(t) = \sum_{i=1}^N r_i^j(t) \tag{5}$$

$$r(t) = \left[ \sum_{j=1}^M \sum_{i=1}^N b^j \cdot c_i^j \cdot h_i(t) \cdot h_i^*(t) \cdot \cos(w_i t + \theta_i) + \sum n_i(t) \right] \tag{6}$$

$$r_i^j(t) = A_0 b^j c_i^j h_i(t) \cdot h_i^*(t) \cdot \cos(w_i t + \theta_i) \tag{7}$$

where,  $i=1:N$ ;  $j=1:M$

$A_0$  is the amplitude of received signal,  $M$  is number of antennas,  $n(t)$  is the Additive White Gaussian Noise(AWGN) component in the propagation path.

Now the received signals at different branches of the receiver can be represented as:

$$y_i^j = r_i^j(t) \cos(w_i t + \Delta w) \tag{8}$$

$$y_i^j = A_0 b^j c_i^j h_i(t) \cdot h_i^*(t) \left[ \frac{1}{2} \cdot 2 \cos(2w_i t + \Delta w) + \cos(\Delta w) \right] \tag{9}$$

$$v_i^j = A_0 b^j c_i^j h_i(t) \cdot h_i^*(t) \left[ \frac{1}{2} \cos(2w_i t + \Delta w) + \cos(\Delta w) \right] \times c_i^j \tag{10}$$

$$v_i^j = A_0 b^j |c_i^j|^2 |h_i(t)|^2 \times \left[ \frac{1}{2} \cos(2w_i t + \Delta w) + \cos(\Delta w) \right] \tag{11}$$

$$v_{0i}^j = A_0 b^j |c_i^j|^2 |h_i(t)|^2 \times \left[ \frac{1}{2} \cos(\Delta w) + n_0(t) \right] \tag{12}$$

where  $\Delta w$  is the frequency difference between the sub-carriers.

Now after applying MRC the signal is,



$$v_{0i}^1 = A_0 b^1 |c_i^1|^2 |h_i(t)|^2 \times \left[ \frac{1}{2} \cos(\Delta\omega) + n_0(t) \right] \quad (13)$$

So, Signal to Interference plus Noise Ratio (SNIR) can be expressed as:

$$S_{0i}^1 = \gamma_{0i}^1 = \frac{|v_{0i}^1|^2}{\sigma_n^2 + \sigma_M^2} \quad (14)$$

where,

Variance of MAI is  $\sigma_M^2$  and  $\sigma_n^2$  is noise variance.

$$S_{0i}^1 = \frac{[A_0 b^1 |c_i^1|^2 |h_i(t)|^2 \times \frac{1}{2} \cos(\Delta\omega)]^2}{|\sigma_n^2 + \sigma_M^2|} \quad (15)$$

$$S_0 = \gamma_{0i}(\Delta\omega) = \frac{\left(\frac{A_0^2}{\sigma_n^2}\right) \times |h_i|^2 \times \frac{1}{4} \cos^2 \theta}{1 + \frac{\sigma_M^2}{\sigma_n^2}} \quad (16)$$

$$S_0 = \gamma_{0i}(\Delta\omega) = \gamma_c(\Delta\omega) (h_i^2) \quad (17)$$

where,

$\gamma_{0i}(\Delta\omega) = \chi^2$  distributed. So the probability density function (PDF) of  $\gamma_{0i}$  can be expressed as:

$$\gamma_{0i} = \frac{\gamma_0^{L-1} e^{-\frac{\gamma_0}{\Gamma}}}{(L-1)! (\Gamma)^L} \quad (18)$$

Where,

$$\Gamma = \gamma_c(\Delta\omega) 2\sigma_\alpha^2 \quad (19)$$

$$S_0 = \frac{\gamma_b \times \left|\frac{h_i}{4}\right|^2 \times \cos^2 \theta}{1 + \xi} \quad (20)$$

Here  $\xi$  is Interference to Noise Ratio (INR) and is assumed to be a zero mean Gaussian random variable variance with  $\sigma^2 = 2 f T_b$ . Which represents the phase jitter due to frequency jitter introduced by the channel.

Now, bit error rate for BPSK modulation signal is

$$B_i(\gamma_{01} f) = \frac{1}{2} \text{erfc} \left[ \frac{\sqrt{\gamma_{01} f}}{2} \right] \quad (21)$$

$$B_i(f) = \frac{1}{2} \text{erfc} \left[ \frac{\sqrt{\gamma_{01} f}}{2} \right] P(\gamma_{01} f) d(\gamma_{01} f) \quad (22)$$

$$B_i = \int B_i(f) P(f) d(f) \quad (23)$$

$$B = \frac{1}{L} \sum_{i=1}^L B_i \quad (24)$$



IV. RESULT AND DISCUSSIONS

In this section, the performance result of a MC-CDMA system are evaluated numerically through Matlab simulation based on the theoretical analysis described in section-III. Performance result are evaluated in terms of BER without and with frequency jitter considering Rayleigh fading channel. BER performance curve evaluated by considering various system parameters such as of INR, processing gain, order of Rayleigh fading etc. The parameters used for numerical computation as shown in Table 1:

Table 1. Table of System Parameters

Parameters	Values
Interference to Noise Ratio (INR)	1-20
SNR input	12,14,16,18,20,22
Phase Jitter (Variance) $\sigma^2$	0.001,0.005,0.01,0.5,0.1,1
Number of Receive Antenna	1-M

Following the theoretical analysis shown in section III, we evaluated the input SNR and output SNR considering the other parameter constant and it is observed that input and out are rational. Figure 3 shows that SNR in (dB) versus SNR out (dB) for MC-CDMA wireless communication system where with the increase of user, SNR is also increased.

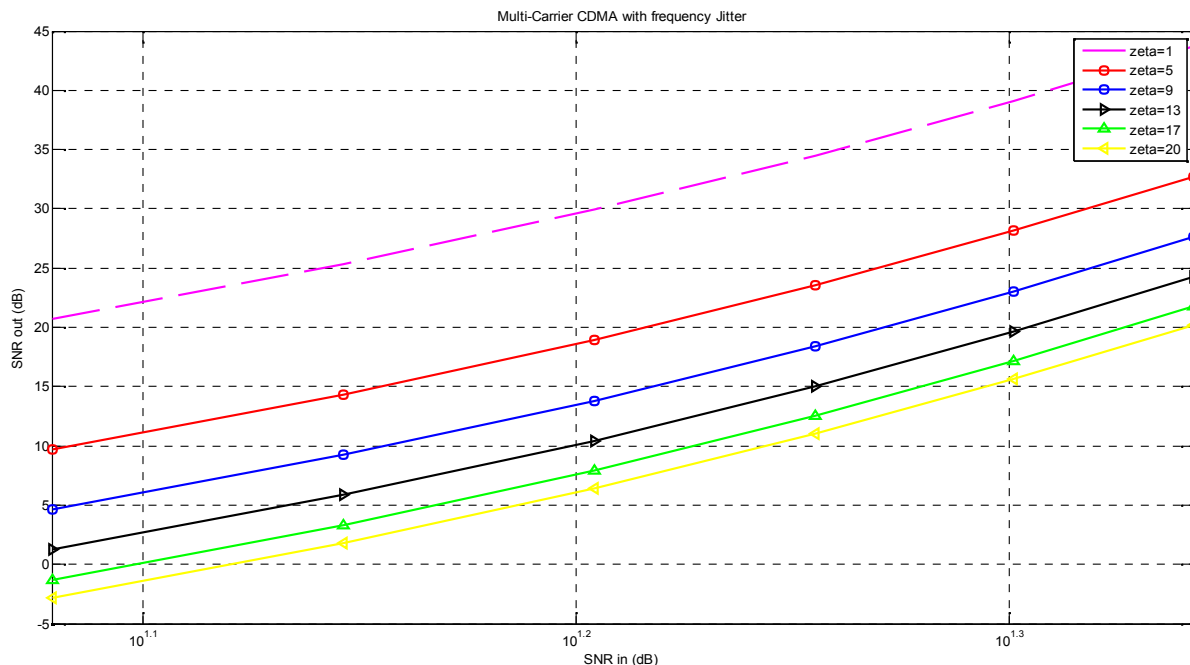
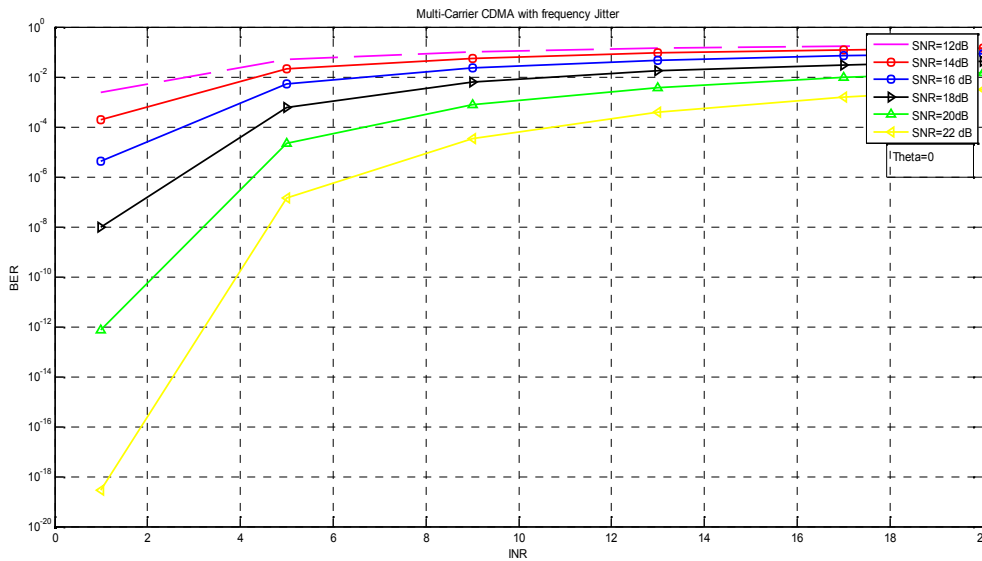


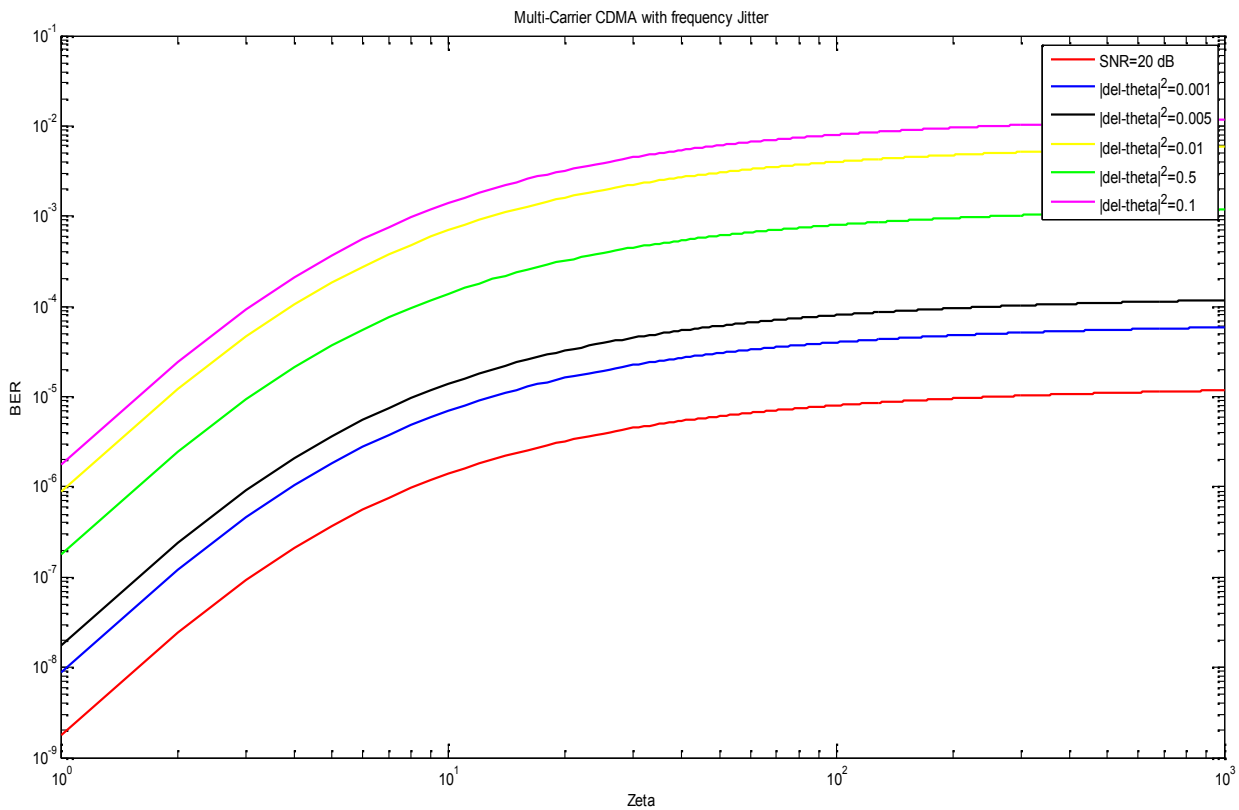
Fig. 3. Plots of SNR in (dB) versus SNR out (dB) for MC-CDMA wireless communication system without Frequency Jitter and with different INR, number of sub carrier N=6.

The plots of BER curve for values of INR 1,5,9,13,17 and 20 verses SNR input 12, 14,16,18,20 and 22 dB are shown in Fig.4. It has been observed that without frequency jitter BER increased rationally with the increase of Noise to Interference Ratio.



**Fig. 4.**Plots of bit error rate (BER) versus INR for MC-CDMA wireless communication system without frequency jitter for different input SNR.

The plots of BER versus INR curve in different frequency jitter environment shown in figure 5. It has been observed that with frequency jitter BER increased more abruptly with respect to the INR.



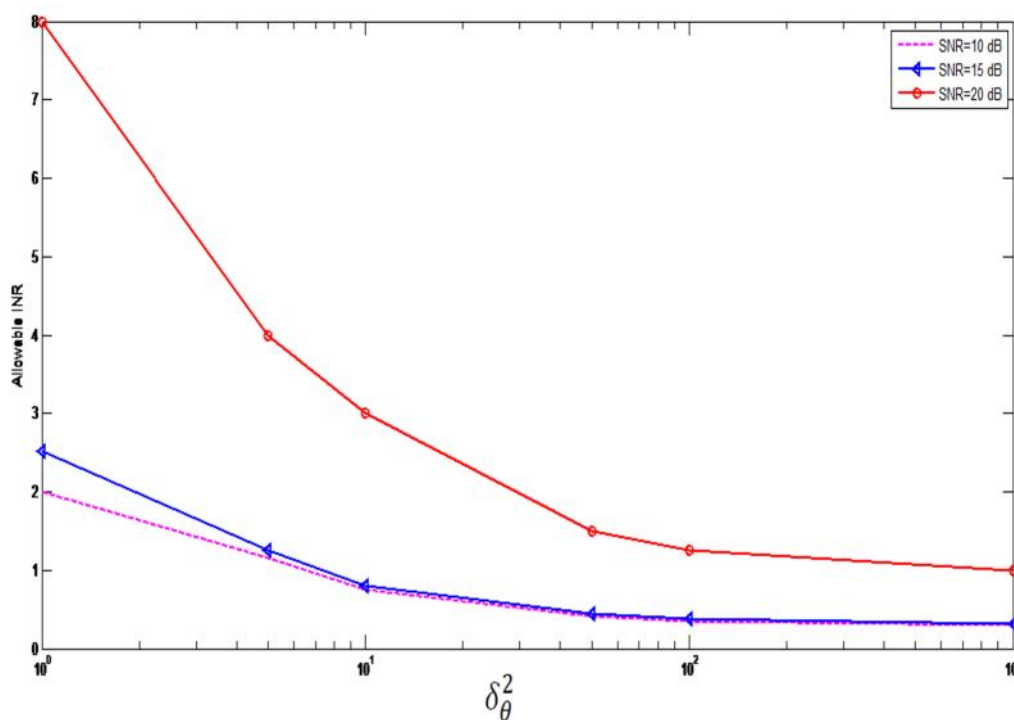
**Fig. 5.** Plots of bit error rate (BER) versus Zeta for MC-CDMA wireless communication system with frequency jitter environment.

From figure 5 we can get following corresponding values at BER $10^{-6}$  :

**Table 2. Table of System Parameters**

VARIANCE, $\sigma^2$	INR- $\xi$ , (SNR=20 dB)	INR- $\xi$ , (SNR=15dB)	INR- $\xi$ ,(SNR=10dB)
0.001	7.9433	2.511	0.7943
0.005	3.9811	1.2589	0.3891
0.01	2.5119	0.7943	0.2512
0.5	1.4125	0.4467	0.1413
0.1	1.2023	0.3802	0.1202
1	1	0.3162	0.1023

Frequency jitter versus allowable user curve in 20 dB SNR, 15 dB SNR and 10 dB SNR shown in Figure 6. Here it is found that with the increase of user, frequency jitter is decreasing.



**Fig. 6. Plots of Allowable INR versus Phase Jitter Variance for MC-CDMA wireless communication system for SNR=20 dB, 15 dB and 10 dB.**

### V. CONCLUSION

An analytical approach is developed to find the bit error rate (BER) performance of a MC-CDMA system considering frequency jitter and OFDM sub-carriers. The analysis is also carried out for multiple receivers and BER performance results are evaluated numerically for several system parameters including MRC and EGC for single antenna. There is significant reduction in INR with increase in variance of frequency jitter.

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